Extended Mixing-Length Applications to Compressible Turbulent Boundary Layers

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Abstract

NUMERICAL calculations based on the compressible boundary-layer equations and an integral form of the kinetic-energy-of-turbulence (IKET) equation are presented for a variety of conditions. The addition of the IKET equation permits the streamwise computation of an additional dependent variable normally taken as an empirical constant in conventional mixing-length formulations. This so-called extended mixing-length hypothesis is not new, having been developed originally by McDonald and Camarata[1] and applied by Chan.[2] Examples given include relaminarization, adverse and favorable pressure gradients, acoustic-energy-induced transition, and surface roughness. The extended mixing-length hypothesis is shown to be considerably more flexible than conventional mixing-length turbulence models.

Contents

Theoretical Method

The extended mixing-length hypothesis represents a turbulence modeling approach that lies between the conventional mixing-length formulations (zero-equation model of turbulence) and the one-equation turbulence models. The IKET equation was obtained by integrating across the boundary layer an equation for the transport of turbulent kinetic energy. The resulting IKET equation contains a source term representing the absorption of incident acoustic energy, the driving force for the transition process. A two-layer model of the turbulent boundary layer was adopted following the classical inner-outer region approach in which separate functional relationships are prescribed in each region, with continuity of the functions between each region. The inner-region damping expressions were taken from Wolfshtein[3] and are similar in form to the well-known expression of Van Driest. In this analysis, the ratio of the outer-region length scale to the boundary-layer thickness (λ/δ) was the additional parameter calculated by streamwise solution of the IKET equation. To improve the fidelity of the IKET analysis, the values of some turbulence model constants were altered to reflect roughness and pressure gradient effects. The alterations can be expressed consistently in terms of κ, the von Kármán constant, and A^+_{eff}, the effective value of the van Driest damping constant in the inner region.

Pressure gradient effects on A^+_{eff} and κ were assessed using equilibrium expressions in conjunction with a lag analysis. Roughness was simulated by considering the classical transitional/fully rough regimes (defined by a roughness Reynolds number) with modifications made to A^+_{eff} and κ to reflect transitional roughness, whereas the fully rough regime was calculated using the form drag of individual roughness elements. Complete details and appropriate credits and references are given in the backup paper, as is a complete nomenclature.

Results

The prediction of relaminarization phenomena is one of the most stringent tests of the innate "physics" of turbulence models. Presented in Fig. 1 are calculated results from the present IKET analysis, the experimental data of Nash-Webber,[4] and calculated laminar and fully turbulent results. The present IKET results are in good agreement with the experimental data. The local momentum thickness Reynolds number does not attain a laminar value until $x \sim 35$ in., which is well after the local skin friction attains its laminar value ($x \sim 31$ in.). Thus the present IKET analysis predicts a profile relaxation effect of the boundary-layer upstream history, a feature not possessed by conventional mixing-length analyses.

One of the best experimental studies concerning compressible turbulent boundary-layer flows under nonequilibrium conditions is that reported by Lewis et al.[5] Data were taken on a wind-tunnel model that consisted of two parts: an outer hollow shell whose walls could be cooled, and an inner pressure-generating body contoured such that both compression and expansion regions were imposed on the outer shell. Comparisons of the IKET calculations and the experimental data are presented in Fig. 2. The variation of A^+_{eff} with pressure gradient using the approach discussed earlier resulted in noticeably better IKET calculations than the assumption of $A^+_{\text{eff}} = 26.0$ irrespective of the pressure gradient. The IKET computed values of the hot- and cold-wall outer-region length scales are shown in Fig. 2, along with the nominal value of 0.09 often used in conventional mixing-length analyses.

Any analytical model of the boundary-layer transition process under hypersonic conditions must include the

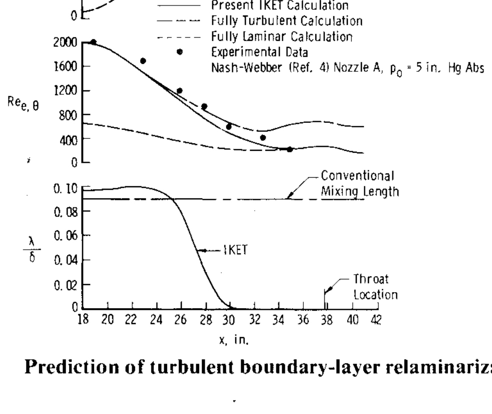

Fig. 1 Prediction of turbulent boundary-layer relaminarization.

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Index categories: Boundary Layers and Convective Heat Transfer – Turbulent; Computational Methods.

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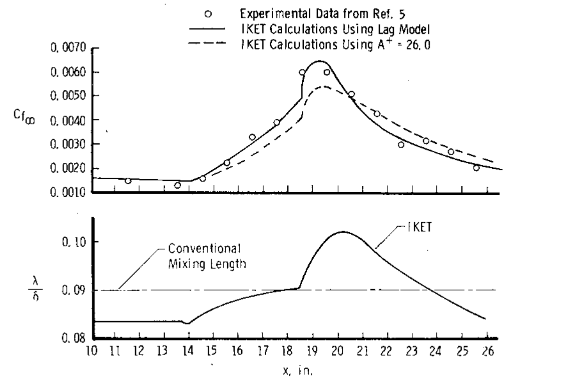

Fig. 2 Skin friction and length scale distributions for adverse/favorable pressure gradient flow.

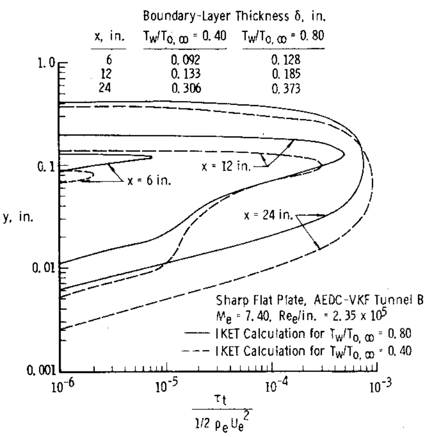

Fig. 3 Prediction of the precursor effect.

precursor effect. Since turbulent bursts are observed far upstream of the conventional transition location where the wall heat transfer or skin friction deviates from the laminar values, transition at hypersonic speeds must occur initially away from the wall. Thus, the outer portion of the mean flow profiles already may be distorted by turbulence effects at the conventional transition location. The precursor effect predicted by the IKET analysis is illustrated in Fig. 3, which gives the turbulent shear stress profiles at three streamwise locations along a sharp flat plate. This calculation was made using the value of absorbed acoustic energy which placed the end of transition at the experimentally determined location. The IKET analysis predicts at the 6-in. location, which is far upstream of the surface-indicated transition location, a sharp "spike" in the turbulent shear stress near the outer edge of the boundary layer. The transitional turbulent shear stress profile at $x = 12$ in. approaches the fully turbulent value at the $x = 24$ in. location, especially for the hot-wall condition. Thus, in the outer region of the boundary layer at $x = 12$ in., the flow is turbulent, and hence the outer portion of the boundary-layer mean flow profiles is distorted by upstream turbulence effects.

Roughness effects were investigated using as a basis of comparison the results of Reda et al.,[6] which were taken on the flat nozzle wall of a supersonic nozzle in which the opposite wall was contoured. Presented in Fig. 4 are the IKET-computed skin-friction coefficients for smooth and 80-, 50-, and 24-grit sand grain roughness, as well as the corresponding

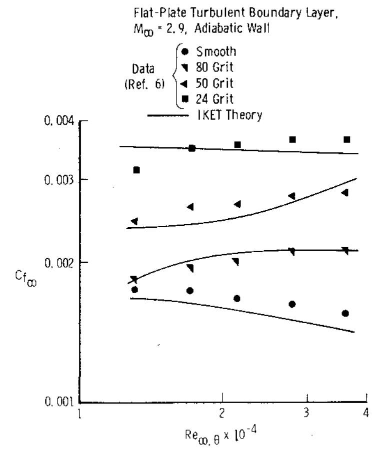

Fig. 4 Skin friction for various wall roughnesses.

experimental data. The 80-grit data are quite interesting in that initially the roughness Reynolds number was very close to the "smooth" limit; hence the resulting nearly smooth-wall value of skin friction. As the Reynolds number increased, the roughness became more pronounced with respect to the viscous sublayer, causing a thinning that is reflected in the higher value of skin friction. The 50-grit roughness at the low freestream Reynolds numbers was large enough so that the viscous sublayer was destroyed completely at all of the freestream Reynolds numbers of interest, with the flow fully rough.

The IKET-based extended-mixing-length hypothesis has been demonstrated to be an effective technique for the calculation of many different compressible turbulent boundary-layer flows. The ability to calculate the outer-region length scale has been shown to be important for flows with significant pressure gradients and roughness and necessary for flows undergoing transition and relaminarization. For situations in which the applicability of conventional mixing-length analyses is questionable or doubtful, the extended mixing-length approach provides a next logical degreee of sophistication.

References

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